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A Boundary that Sustains a Negligible Specular Reflection Coefficient Over a Wide Frequency Band

by

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Abstract

In a previous paper the authors analyzed and discussed the specular reflection coefficient of a plane boundary comprising a plate, a compliant layer and a fluid. The analysis showed that a negligible specular reflection coefficient may be derived provided specific resonance conditions are met. The resonance of concern is that between the surface mass of the plate and the surface stiffness of the compliant layer. The conditions of resonance included the value that must be assigned to the loss factor in the compliant layer. In the present report, an attempt is made to determine the conditions that must be placed on the surface stiffness of the compliant layer in order to increase the frequency range over which a negligible specular reflection coefficient may be maintained. The tolerances in these conditions are also estimated.

Introduction

The specular reflection coefficient $R(\underset{\sim}{k},\omega)$ of a plane surface that interfaces with a semi-infinite fluid atop and that possesses a uniform surface impedance $Z(\underset{\sim}{k},\omega)$, is familiarly given by

$$R(\underset{\sim}{k},\omega) = [Z(\underset{\sim}{k},\omega)-Z_1(\underset{\sim}{k},\omega)][Z(\underset{\sim}{k},\omega)+Z_1(\underset{\sim}{k},\omega)]^{-1} \;\; ; \;\; \underset{\sim}{k} = \{k, k_y\} \quad , \qquad (1)$$

where $Z_1(\underset{\sim}{k},\omega)$ is the surface impedance of the fluid on the plane, $(\underset{\sim}{k})$ is the wavevector variable in the plane, and (ω) is the frequency variable; see Fig. 1 [1]. The surface impedance of the fluid is expressed in the form

$$Z_1(\underset{\sim}{k},\omega) = (\rho c / \bar{k}_3) \quad , \qquad (2a)$$

$$\bar{k}_3(\underset{\sim}{k},\omega) = (1-\gamma^2)^{1/2}U(1-\gamma^2) - i(\gamma^2-1)^{1/2}U(\gamma^2-1) \;\; ; \;\; \gamma = (|\underset{\sim}{k}| c/\omega) \quad , \qquad (2b)$$

where (ρ) and (c) are the density and speed of sound in the fluid, respectively, U is the step function, and $\{\underset{\sim}{k},\omega\}$ defines the incidence on and the specular reflection from the plane boundary; again, see Fig. 1 [1]. From Eqs. (1) and (2a) one obtains

$$R(\underset{\sim}{k},\omega) = [\bar{Z}(\underset{\sim}{k},\omega)\bar{k}_3 - 1]\;[\bar{Z}(\underset{\sim}{k},\omega)\bar{k}_3 + 1]^{-1} \;\; ; \;\; \bar{Z}(\underset{\sim}{k},\omega) = [Z(\underset{\sim}{k},\omega)/(\rho c)] \quad . \qquad (3)$$

The normalized surface impedance $\bar{Z}(\underset{\sim}{k},\omega)$ is a complex quantity that can be expressed in the form

$$\overline{Z}(\underset{\sim}{k},\omega)=\overline{Z}_R(\underset{\sim}{k},\omega) + i\overline{Z}_I(\underset{\sim}{k},\omega) \quad , \tag{4}$$

where $\overline{Z}_R$ and $\overline{Z}_I$ are real quantities and for the structural system to be stable it requires that

$$\overline{Z}_R(\underset{\sim}{k},\omega) + \text{Re}\{(\overline{k}_3)^{-1}\} > 0 \quad . \tag{5}$$

In this connection it is observed, from Eq. (2b), that the normalized surface impedance $(\overline{k}_3)^{-1}$ of the fluid on the plane is either wholly real or wholly imaginary. A real $(\overline{k}_3)^{-1}$ defines the supersonic spectral range; i.e., the range $\gamma < 1$, and an imaginary $(\overline{k}_3)^{-1}$ defines the subsonic spectral range, i.e., the range $\gamma > 1$. In the vicinity of $\gamma = 1$, $|(\overline{k}_3)^{-1}|$ is large, at $\gamma = 1$, $|(\overline{k}_3)^{-1}|$ is singularly large. A propagating incidence is commensurate with a normalized surface fluid impedance that is wholly real and, therefore, may be cast in the form

$$[\overline{Z}_1(\underset{\sim}{k},\omega)]^{-1} = \overline{k}_3(\underset{\sim}{k},\omega) = \cos\theta \; ; \; \gamma < 1 \; ; \; 0 \leq \theta < (\pi/2) \quad , \tag{6a}$$

$$(\underset{\sim}{k}\, c/\omega) = \{\sin(\theta)\cos(\phi), \; \sin(\theta)\sin(\phi)\} \quad , \tag{6b}$$

where $\{\theta, \; \phi\}$ is the angular vector of incidence; see Fig. 1. To obtain a substantially negligible specular reflection coefficient; namely

$$R(\underset{\sim}{k},\omega) = (\varepsilon/\sqrt{2})[1+\varepsilon+(\varepsilon^2/2)]^{-(1/2)} \; ; \; |\varepsilon| << 1 \quad , \tag{7}$$

the following conditions need to be satisfied simultaneously

$$\bar{Z}_R(\underset{\sim}{k},\omega)\cos(\theta) = (1+\varepsilon) \; ; \; \bar{Z}_I(\underset{\sim}{k},\omega)\cos(\theta) = \varepsilon \; ; \; \gamma < 1 \; , \tag{8}$$

where the smallness of $|\varepsilon|$, as compared with unity, is yet to be specified.

In general, designing a boundary which presents a given mechanical surface impedance to the fluid atop is termed "conditioning". Thus, it is common to call a boundary that incorporates merely a plate, a "conditioning plate." The plate may be generalized into a basic boundary where this boundary possesses a mechanical surface impedance that roughly relates to that of a plate; e.g., a membrane. Situations arise in which a more compounded boundary needs to be designed. A member in a class of such boundaries may be one for which the boundary is designed to possess a mechanical surface impedance that largely matches that of the surface impedance of the fluid in the plane of the boundary. Often a matching of this kind requires a resonance to occur within the boundary. Another member, in this class of boundaries, may be one for which the boundary is designed to resonate with the fluid atop; i.e., the surface impedance of the boundary with that of the fluid in the plane of the boundary describes a resonant dynamic system. To satisfy either of the resonances, elemental surface impedances that are compliant layers must be incorporated into the composition of the boundary. A boundary that is designed to accommodate one or the other of these resonances by incorporating compliant layers is called, therefore, "conditioning compliance." A resonance usually defines and extends over a narrow frequency band that is centered on the resonance frequency. If the advantages, that the resonance bestows upon the conditioning compliance that it serves, are to be sustained over a wider and wider frequency band, means to maintain the resonance conditions, over such a frequency bandwidth, must be devised. Indeed, establishing techniques and mechanisms for

implementing a wide frequency band conditioning compliance has been actively pursued; e.g., an implementation of this kind had been proposed and analyzed by Sheiba and Colleagues at EG&G during the late 1980's and early 1990's [2]. Usually these techniques and mechanisms call upon an elaborate compliant layer that is placed on a basic boundary, a compounded combination of surface stiffnesses and surface masses, and/or, in addition, an inclusion of even non-mechanical elements. In this report the simpler mechanism is examined: A compliant layer that is placed on an initial boundary classified as a conditioning plate; in this arrangement the top surface of the compliant layer faces the fluid. The mechanism for achieving a wide-frequency-band conditioning compliance is then associated with rendering the surface stiffness of the compliant layer frequency dependent. The analysis of this frequency dependence allows one to demonstrate, in a straightforward manner, the conditions, and the tolerances on these conditions, that need to be satisfied in order to achieve the desired boundary. Although this approach is of limited scope, it addresses questions that are relevant to any other attempt to achieve a wide-frequency-band conditioning compliance.

Nearly a decade ago the authors discussed the criteria that ensures a negligible specular reflection coefficient for a boundary that incorporates a panel with a compliant layer atop; the fluid then lies atop this compliant layer [1]. The criteria stated in Eq. (7) are compatible with those stated in Reference 1. The negligible specular reflection coefficient is achieved, in this reference, only over a narrow frequency-band (and/or over a narrow angular-band). The purpose of the present report is to decipher conditions that may be placed on a boundary, comprising essentially a panel and a compliant layer, that will ensure a negligible specular reflection coefficient over a wider frequency-band (and/or over a wider angular-band). Recently, a corresponding attempt was undertaken with respect to dynamic absorbers. In this attempt a

number of sprung masses, with a wide distribution of resonance frequencies, replaces the single sprung mass [3,4]. In an analogous manner, one may then analyze and discuss the implications involved in rendering a boundary that passively maintains these wider frequency bandwidths. Moreover, one may even address as to whether an actively controlled boundary may be devised with the appropriate properties to maintain these wider frequency bandwidths.

I. Normalized Surface Impedance of a Boundary Comprising a Panel and a Compliant Layer

The elements of a fluid-loaded boundary of this kind are depicted in Fig. 2a and the equivalent circuit diagram is shown in Fig. 2b [1]. An isotropic panel is characterized by the normalized surface impedance

$$\overline{Z}_2(\underset{\sim}{k},\omega) \;=\; i\omega\overline{M}\,\overline{Z}_g(\underset{\sim}{k},\omega) \;=\; i(\omega/\omega_c\varepsilon_c)\overline{Z}_g(\underset{\sim}{k},\omega)\;;\;\; \overline{M}=[M/(\rho c)]\;, \tag{9a}$$

$$\overline{Z}_g(\underset{\sim}{k},\omega) \;=\; [(1-g)-i\eta]\;;\;\; \eta=[\eta_2+(g\eta_p)]\;;\;\; g=(1+\eta_2^2)(|\underset{\sim}{k}|/k_p)^n$$

$$\text{and if } n=4,\; k_p^2=(\omega\omega_c/c^2)\quad , \tag{9b}$$

$$\overline{M} \;=\; (1+\eta_2^2)^{-1}\overline{M}_2\;;\;\; \overline{M}_2=[M_2/(\rho c)]\;;\;\; \varepsilon_c=(\omega_c\overline{M})^{-1}\quad ;$$

$$\varepsilon_2 \;=\; (\omega_c\overline{M}_2)^{-1} \;=\; (1+\eta_2^2)^{-1}\varepsilon_c\;;\;\; \eta=(\eta_2+g\eta_{p)}\quad , \tag{9c}$$

where (M_2) is the surface mass, (k_p) is the free wavenumber, (ω_c) is the critical frequency with respect to the speed of sound (c) in the fluid, (ε_2) is the fluid loading parameter, (n) is the flexural index $[(n)$ is equal either to (2) or $(4)]$, $(g\eta_p)$ and (η_2) are the loss factors associated with the surface stiffness and the surface mass of the panel, respectively, and it is to be understood that the explicit dependence of quantities and parameters on the vector variable $\{\underset{\sim}{k},\omega\}$ may, at times, be omitted as obvious; e.g., $g=g(\underset{\sim}{k},\omega)$ and $\eta_p=\eta_p(\underset{\sim}{k},\omega)$ in the above equations. Were the panel orthotropic a replacement in the manner

$$(|\underset{\sim}{k}|/k_p)^n(1+i\eta p) \to [(k/k_p)^2(1+i\eta_{px})+(k_y/k_{py})^2\ (1+i\eta_{py})]^{(n/2)} \quad , \tag{9d}$$

must be introduced in Eq. (9b) [5]. In Eq. (9d), $\{k_{px},\ k_{py}\}$ and $\{\eta_{px},\ \eta_{py}\}$ are the free wavevector and the loss vector, respectively. Similarly, if $n = 2$, rather than $n = 4$, the free wavenumber (k_p) assumes the simpler form

$$k_p = (\omega / c_p) \text{ with } c_p \text{ independent of } (\omega) \quad , \tag{9e}$$

and (ω_c) becomes merely a suitable normalizing frequency. For the sake of simplicity the panel is considered to be an isotropic plate so that $(c_p / c) = (\omega / \omega_c)^{1/2}$ and $n = 4$ reign; notwithstanding that a situation may exist in which the employment of orthotropic panels, that are or are not plates, may prove beneficial.

The compliant layer is characterized by the normalized surface impedance

$$\bar{Z}_{12}(\underset{\sim}{k},\omega) = (\bar{K}/i\omega)(1+i\eta_{12}) \ ; \quad f = (\bar{K}_{12}/\bar{K}) \ ; \quad \omega_1^2 = (\bar{K}_{12}/(\bar{M}_{12}) \quad , \tag{10a}$$

$$\bar{f} = (\omega/\omega_o)^2 f \ ; \ \omega_o^2 = (\bar{K}_{12}/\bar{M}) = \omega_1^2(1+\eta_2^2) \ ; \ \bar{K}_{12} = [K_{12}/(\rho c)] \quad , \tag{10b}$$

where, again, the surface impedance of the compliant layer is considered to be isotropic, (K_{12}) is a constant (independent of $\{\underset{\sim}{k},\omega\}$) surface stiffness, (η_{12}) is the loss factor associated with the compliant layer, and (f) is a dimensionless and a real function of the normalized frequency (ω/ω_o); namely, $f = f(\omega/\omega_o)$. The normalizing frequency (ω_o) is defined in Eq. (10). The factor $(f)^{-1}$ is a modification parameter that defines the actual surface

stiffness (K) of the compliant layer in terms of the constant surface stiffness (K_{12}), again, as defined in Eq. (10). In this context, $(\bar{f})^{-1}$ is termed the modification factor and $(\bar{f})$ is, then, the inverse modification factor.

The normalized surface impedance $\overline{Z}\{\underset{\sim}{k},\omega\}$, of the boundary, that is sketched in Fig. 2a, is

$$\overline{Z}(\underset{\sim}{k},\omega) = \overline{Z}_{12}(\underset{\sim}{k},\omega)\overline{Z}_2(\underset{\sim}{k},\omega)[\overline{Z}_{12}(\underset{\sim}{k},\omega)+\overline{Z}_2(\underset{\sim}{k},\omega)]^{-1} \quad , \tag{11}$$

as can be verified with the help of Fig. 2b. Substituting Eqs. (4), (9) and (10) in Eq. (11) one obtains

$$\overline{Z}(\underset{\sim}{k},\omega) = (i\omega\overline{M})(\overline{Z}_g)(1+i\eta_{12})[(1+i\eta_{12})-\bar{f}\,\overline{Z}_g]^{-1} \quad , \tag{12a}$$

or equivalently

$$[\overline{Z}_R\cos(\theta)] = B_c(\omega,\theta)[\eta_{12}\bar{f}\,|\overline{Z}_g|^2 + \eta(1+\eta_{12}^2)]/A \quad , \tag{12b}$$

$$[\overline{Z}_I\cos(\theta)] = B_c(\omega,\theta)[(1+\eta_{12}^2)(1-g)-\bar{f}\,|\overline{Z}_g|^2]/A \quad , \tag{12c}$$

where

$$B_c(\omega,\theta) = (\omega / \omega_c)[\cos(\theta) / \varepsilon_c] \quad , \tag{13a}$$

$$A \;=\; |(1+i\eta_{12}) - \bar{f}\,\bar{Z}_g|^2 \quad , \tag{13b1}$$

which, in turn, may be cast either in the form

$$A \;=\; [\{1-\bar{f}(1-g)\}^2 + \{\eta_{12}\bar{f}\eta\}^2] \quad , \tag{13b2}$$

or, equivalently, in the form

$$A \;=\; [(1+\eta_{12}^2) + \bar{f}^2 |\bar{Z}_g|^2 \;-\; 2\bar{f}\{(1-g) - \eta_{12}\eta\}] \quad ,\tag{13b3}$$

$$\bar{Z}(\underset{\sim}{k},\omega) \;=\; \bar{Z}_R(\underset{\sim}{k},\omega) + i\bar{Z}_I(\underset{\sim}{k},\omega) \;\; ; \quad \bar{Z}_g^* \;=\; [(1-g)+i\eta] \;\; ;$$

$$|\bar{Z}_g|^2 = [(1-g)^2 + \eta^2] \quad . \tag{13c}$$

As already indicated in Eqs. (12b) and (12c) and hereafter, quantities that are dependent on $\{\underset{\sim}{k}, \omega\}$, but are to be evaluated in the supersonic range only, are to be stated free of this explicit dependence. Thus, for example

$$R(\underset{\sim}{k},\omega) \equiv R\{(\omega / c)\sin(\theta)\cos(\phi),\, (\omega / c)\sin(\theta)\sin(\phi),\;\; \omega\} \rightarrow R \;. \tag{6c}$$

Subjecting Eq. (12) to the conditions stated in Eq. (8) and imposing, in addition, that $\varepsilon = 0$, one obtains

$$[Z_R \cos(\theta)]_o \to [B(\omega,\theta) \,|\, \bar{Z}_g \,|^2 \,\{\eta_{12}(1-g)+\eta\}^{-1}]_o = 1 \quad , \tag{14a}$$

$$[Z_I \cos(\theta)]_o \to 0 \;\; ; \;\; [(1+\eta_{12}^2)(1-g) \;=\; \bar{f} \,|\, \bar{Z}_g \,|^2]_o \quad , \tag{14b}$$

$$[A]_o \to [\bar{f}(1-g)^{-1}\{\eta_{12}(1-g)+\eta\}^2]_o \quad . \tag{14c}$$

The brackets $[\cdots]_o$ evaluate the enclosed quantity with $\varepsilon = 0$. [cf. Eq. (8).] Were Eq. (14) satisfied, the specular reflection coefficient R, stated in Eq. (3) with Eq. (6) imposed, would be equal to zero. Thus, Eq. (14) constitutes the design criteria for achieving a boundary of negligible specular reflection coefficient. If Eq. (14) is only nearly satisfied, the specular reflection coefficient R, stated in Eq. (3) with Eq. (6) imposed, would deviate from its value of zero. One may then request that the deviation in $|R|$ does not exceed a predetermined value and request the limits on the parametric variations that would ensure that this predetermined value for $|R|$ would not be exceeded. These parametric variations constitute the design criteria tolerances for the construction of a surface for which the absolute values for the specular reflection coefficient remain tightly bounded within these preset values.

II. Designing a Negligible Specular Reflection Coefficient for the Boundary Specified in Section I

To establish an appropriate connection with Reference 1 and to duplicate the results thereof, one needs merely conform some of the quantities and parameters to those used in Reference 1; namely

$$g \to 0 \ ; \ (g\eta_p) \to 0 \ , \ \eta \to \eta_2 \ ; \quad f = 1 \quad . \tag{15}$$

Substituting Eq. (15) in Eq. (14) one immediately recovers Eq. (30) of Reference 1. [Erratum: (m) in Eq. (30c) of Reference 1 needs to be corrected to read (M_2).] In Reference 1 it was found that a negligible specular reflections coefficient, under the conditions stated in Eqs. (7) and (15), is achieved for a frequency (ω) that is very nearly equal to (ω_1) and for an angle (θ) defined by the relationship

$$(\omega / \omega_1)^2 \simeq (1+\eta_{12}^2) \quad ; \quad (\overline{K}_{12} / \omega)\cos(\theta) \simeq (\eta_{12}+\eta_2)(1+\eta_{12}^2)^{-1} \quad ;$$

$$(\omega \overline{M}_2)\cos(\theta) \simeq (\eta_{12}+\eta_2) \quad . \tag{16}$$

[cf. Eq. (30) of Reference 1.]

The purpose in the present report is to examine the parametric values of the boundary that are needed to achieve a negligible specular reflection coefficient over a wider frequency bandwidth and/or a wider angle of incidence than that achieved in Reference 1. To set the stage it may be useful to assume, again, that the surface impedance of the panel is largely surface mass controlled; namely

$$\bar{Z}_2(\underset{\sim}{k},\omega) \to i\omega\bar{M}(1-i\eta_2) \ ; \ g \to 0 \ ; \ g\eta_p \to 0 \ ; \ \eta \to \eta_2 \tag{17a}$$

Substituting Eq. (17a) in Eqs. (12)-(14) yields

$$[Z_R \cos(\theta)] = B_c(\omega,\theta)[\eta_{12}\bar{f}(1+\eta_{12}^2)+\eta_2(1+\eta_{12}^2)]/A \quad , \tag{18a}$$

$$[Z_I \cos(\theta)] = B_c(\omega,\theta)[(1+\eta_{12}^2)-\bar{f}(1+\eta_2^2)]/A \quad , \tag{18b}$$

$$A = [(1+\eta_{12}^2)+(\bar{f})^2(1+\eta_2^2)-2\bar{f}\{1-\eta_{12}\eta_2\}] \quad , \tag{18c}$$

$$[Z_R \cos(\theta)]_o = [B_c(\omega,\theta)(1+\eta_2^2)(\eta_{12}+\eta_2)^{-1}]_o = 1 \quad , \tag{19a}$$

$$[Z_I \cos(\theta)]_0 \to 0 \ ; \ [(1+\eta_{12}^2) = \bar{f}(1+\eta_2^2)]_o \quad , \tag{19b}$$

$$[A]_o \to [\bar{f}\{(\eta_{12}+\eta_2)\}^2]_o \ ; \ \bar{Z}_g^* = (1+i\eta_2) \ ; \ |\bar{Z}_g|^2 = (1+\eta_2^2) \quad . \tag{19c}$$

From Eqs. (9c) and (10b), Eqs. (19a) and (19b) may be cast in the form

$$[(\eta_{12})]_o = [\{B_c(\omega,\theta)(1+\eta_2^2)-\eta_2\}]_o \quad , \tag{20a}$$

$$[(\bar{f})]_o \ = (1+\eta_2^2)^{-1}\{1+[(\eta_{12})]_o^2\} \quad , \tag{20b}$$

where $B_c(\omega,\theta)$ and (η_2) are stated in Eqs. (13a) and (9), respectively, and, again, $[\cdots]_o$ encloses parameters that are evaluated for negligible specular reflection coefficients. For a given boundary (ω_o / ω_c), (ε_c) and (η_2) are assumed to be specified parameters; the variables in Eq. (20) are then (ω / ω_o) and $[\cos(\theta)]$. The loss factor $[(\eta_{12})]_o$ and the inverse modification factor $[(\bar{f})]_o$ are depicted, as functions of (ω / ω_o) and $[\cos(\theta)]$, for several values of (ω_o / ω_c), (ε_c) and (η_2), in Figs. 3 and 4, respectively. For checking purposes, the corresponding absolute values of the specular reflection coefficients $[| R\{(\omega / \omega_o),\ \cos(\theta)\} |]_o$, as functions of (ω / ω_o) and $[\cos(\theta)]$, are depicted in Fig. 5. Figures 3-5 clearly demonstrate that extended regions in which values of $[(\eta_{12})]_o$ and $[(\bar{f})]_o$ can be found and that these values indeed yield, in these extended regions, negligible specular reflection coefficients; $[R]_o \to 0$. Equation (10) reminds one that $[(f)^{-1}]_o$ is the modification parameter in the surface stiffness (K) of a compliant layer that necessarily renders a boundary to be of negligible specular reflection coefficient

$$[K(\omega / \omega_o)]_o \ = \ [\{f(\omega / \omega_o)\}^{-1}]_o \ K_{12} \quad , \tag{21}$$

where K_{12} is a constant independent of frequency; the frequency dependence of $[K(\omega / \omega_o)]_o$ is entirely accounted for by the modification factor $[\{f(\omega / \omega_o)\}^{-1}]_o$. Equation 20, establishes

the relationship between the inverse modification factor $[(\bar{f})]_o$ and the loss factor $[(\eta_{12})]_o$. This relationship is depicted in Fig. 6a for two values of (η_2); $\eta_2 = 10^{-2}$ and 10^{-1}. On the other hand, again using Eq. (20), the dependence of the modification parameter $[(f)^{-1}]_o$ on (ω / ω_o) and $[\cos(\theta)]$ is depicted in Fig. 6b. [cf. Fig 4.] Figure 6 shows that in the lower range of the loss factor in the compliant layer; i.e., when $[(\eta_{12})]_o \leq (1/2)$, $[(f)^{-1}]_o$ is quadradically dependent on the normalized frequency (ω / ω_o) and is largely independent of the angular function $[\cos(\theta)]$. [cf. Fig. 3.] As the loss factor $[(\eta_{12})]_o$ increases into the higher range; i.e., when $[(\eta_{12})]_o \geq (3/2)$, the modification parameter $[(f)^{-1}]_o$ becomes asymptotically independent of the normalized frequency (ω / ω_o), but becomes inversely proportional to $[\cos^2(\theta)]$. In addition, Eq. (20) states that a boundary of negligible reflection coefficient requires the loss factor $[(\eta_{12})]_o$ to be linearly dependent on the normalized frequency (ω / ω_o) and on the angular function $[\cos(\theta)]$. In this connection one realizes that the damping of the boundary is contributed by both, the damping in the panel, as measured by (η_2), and in the compliant layer, as measured by (η_{12}). The combined loss factor $(\eta_{12} + \eta_2)$ may exceed the loss factor necessary to achieve a negligible reflection coefficient. In such a case one may require $[(\eta_{12})]_o$ to be negative. This requirement cannot be achieved passively. Then, to achieve a negligible specular reflection coefficient, a call for an active control surface impedance may become mandatory.

III. Variations of $[(\eta_{12})]_o$ and of $[(\bar{f})]_o$

Although a negligible specular reflection coefficient may be a design goal, often achieving an absolute value for the specular reflection coefficient that is small compared with unity may suffice. Indeed, for many practical purposes a $|R|$ (= $|R(\underset{\sim}{k},\omega)|$ that lies in the supersonic range, where $\gamma = (|\underset{\sim}{k}|c/\omega) < 1$) that is less than one third is satisfactory enough. The extreme of this value indicates and absorption of 90% of the incident spectral density. Of course, the limits on the variation in $|R|$ may be practically induced by the inability to meet the prescribed values of the loss factor $[(\eta_{12})]_o$ and the inverse modification factor $[(\bar{f})]_o$. Using Eqs. (18) and (19) one may derive relationships between the values of $|R|$ and the proportional variations of $[(\eta_{12})]_o$ and of $[(\bar{f})]_o$. These variations and relationships are readily derived to be

$$[(\eta_{12})^{-1}]_o\Delta(\eta_{12}) \;=\; \pm 2|R|[1+(\eta_2/\eta_{12})]_o$$

$$\{[1+2\eta_{12}\eta_2)^{-1}]_o^2 + [2\eta_{12}(1+\eta_{12}^2)^{-1}]_o^2\}^{-(1/2)} \quad , \tag{22a}$$

$$[(\bar{f})^{-1}]_o\Delta(\bar{f}) \;\equiv\; [(\bar{f})]_o\Delta(\bar{f})^{-1} \;=\; \pm 2|R|[1+(\eta_2/\eta_{12})]_o$$

$$\{[\eta_{12}^2(1+\eta_{12}^2)^{-1}]_o\}^{1/2} \quad , \tag{22b}$$

where (Δ) designates a variation in the quantity on which it operates. In Figs. 7 and 8 the ratios $\{|R|[\eta_{12}]_o\}^{-1}\Delta(\eta_{12})$ and $\{|R|[(\bar{f})]_o\}^{-1}\Delta(\bar{f})$ are depicted as functions of $[(\eta_{12})]_o$ and of $[(\bar{f})]_o$, respectively, for values of (η_2) equal to 10^{-2} and to 10^{-1}. It is immediately clear from Figs. 7 and 8 that when $[(\eta_{12})]_o \le 1$ or equivalently when $[(\bar{f})]_o = [(1+\eta_{12}^2)]_o(1+\eta_2^2)^{-1} \le 2$,

the permissible variations of $[(\eta_{12})]_o$ are much more laxed than those of $[\bar{f}]_o$, if reasonable low values are to be achieved in the designed $|R|$; $|R|$, again, is the specular reflection coefficient of the conditioning compliance in the supersonic range of spectral space. [cf. Eqs. (3) - (6).] On the other hand, when $[(\eta_{12})]_o \geq 1$ or equivalently when $[(\bar{f})]_o = [(1+\eta_{12}^2)]_o(1+\eta_2^2)^{-1} \geq 2$, the permissible variations of $[(\eta_{12})]_o$ are comparable to those of $[(\bar{f})]_o$. Moreover, in this range of values the variations of $[(\eta_{12})]_o$ and of $[(\bar{f})]_o$ are comparable to those regarding $|R|$; i.e., if in the supersonic range of spectral space $|R|$ is allowed to reach $(1/3)$, the proportional variations of $[(\eta_{12})]_o$ and of $[(\bar{f})]_o$ are allowed approximately the same reach. The situations just discussed are illustrated in Figs. 9 and 10. In these figures $|R|$ is determined with $(\eta_{12}) = 1.2[(\eta_{12})]_o$ and with $(\bar{f}) = 1.1[(\bar{f})]_o$, respectively. The range in which $|R| < 0.3$ and the range in which $|R| > 0.3$ are separated by a solid line. The variational limits depicted in Figs. 7 and 8 are properly reflective in Figs. 9 and 10, respectively. Whether the maintenance of the parameters $[(\eta_{12})]_o$ and $[(\bar{f})]_o$ can be kept within the required limits, in any practical situation designed to sustain $|R| \leq 0.3$, is yet to be tested. The elements of the design and the form of the testing are, however, in hand, in part, due to this report.

IV. Concluding Remarks

In Reference 1, the sensitivity of $|R|$ to variations in the values of $(\overline{K}_{12} / \omega_c)$ and of (η_{12}) are investigated. In this investigation, $|R|$ is restricted not to exceed (10^{-1}) and $[\varepsilon_c / \cos^2(\theta)]$ is maintained at (10^{-1}). The sensitivity of $|R|$ to these variations seems to be supported by the analysis presented in the preceding section. Indeed, both Figs. 5 and 6 of Reference 1 are so supported by Figs. 9 and 10.

In the same vein, the greater laxity in the sensitivity of the specular reflection coefficient $|R|$ to variations in the values of (η_{12}) as compared to those in the values of $(\bar{f})$ is significant to the design process. If one focuses on the limited range in which $[(\eta_{12})]_o \lesssim 1$ and, therefore, $[(\bar{f})]_o \lesssim 2$, the specification of these parameters become well nigh impractical, especially with respect to the specification of $[(\bar{f})]_o$. The tolerances that are imposed, in the design processes, on the value of $[(\eta_{12})]_o$ and, especially, on the value of $[(\bar{f})]_o$ that lie within the range of these inequalities may thus become too difficult to achieve. Without mentioning specific cases, failure to meet these kind of values and tolerances have already been encountered [6]. Failure of this type and in this range are caused not only as a result of the overly strict tolerances, but also because the dependence of the modification parameter $[(f)^{-1}]_o$ on the normalized frequency (ω / ω_o) is steep; essentially quadratic. The former is illustrated in Figs. 7 and 8 and the latter in Fig. 6. It appears, therefore, that in trying to design a viable conditioning compliance, the range defined by the inequality $[(\eta_{12})]_o \gtrsim 1$ and, therefore, also the inequality $[(\bar{f})]_o \gtrsim 2$, need to be imposed. In this range, not only are the tolerances more reasonable, but the frequency dependence of the modification parameter is more achievable, notwithstanding that the dependence on $[\cos(\theta)]$ is more severe in this range than in the previous lower range, where $[(\eta_{12})]_o \lesssim 1$ and $[(\bar{f})]_o \lesssim 2$. [cf. Figs. 6-8 and remarks post Eq. (21).] Clearly, criteria, and

tolerances on these criteria, that are involved in the design of a compounded format for the conditioning compliance may follow similar treatments with largely similar conclusions.

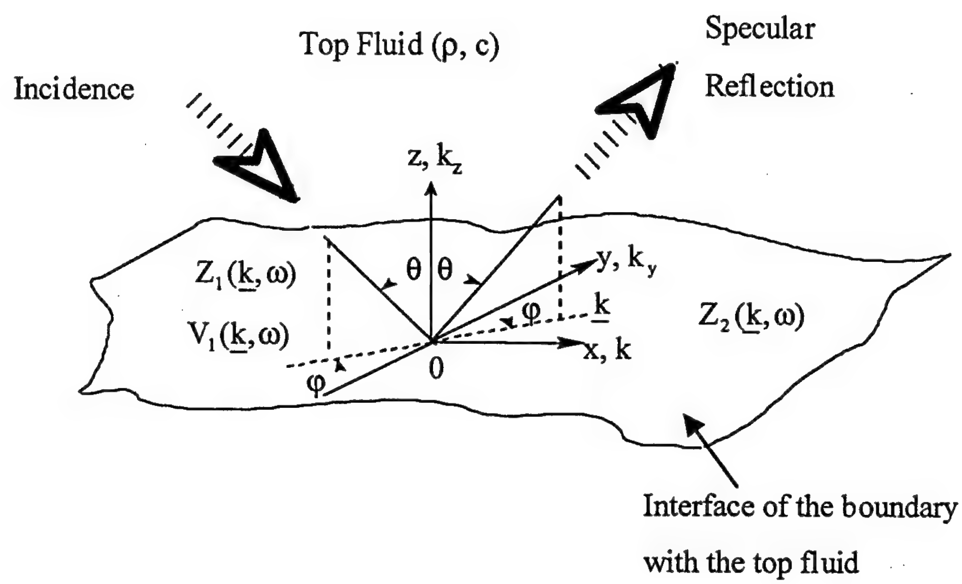
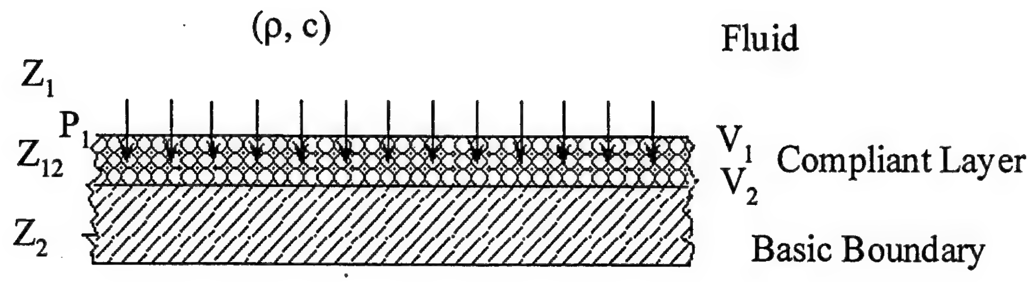
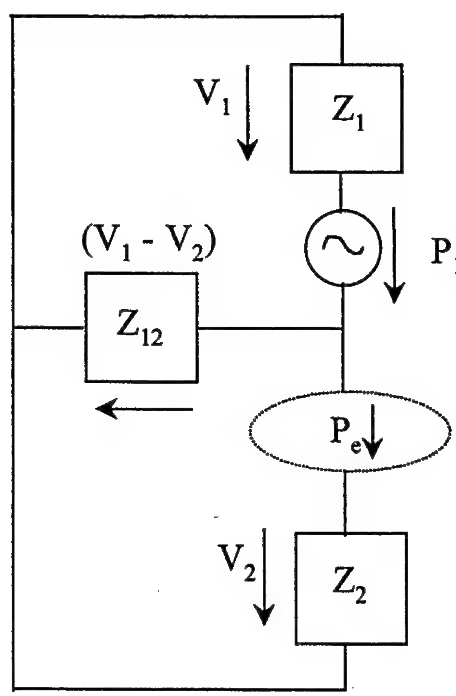


Fig. 1. Incidence and specular reflection as defined by the angular vector $\{\theta, \phi\}$. The velocity $V_1(\underset{\sim}{k}, \omega)$ is on the interface of the boundary and the top fluid. The surface impedance $Z_1(\underset{\sim}{k}, \omega)$ is that of the top fluid and $Z_2(\underset{\sim}{k}, \omega)$ is the surface impedance of the basic boundary; e.g., a backing plate.



a)

Fig. 2a. A boundary composed of a basic boundary and a compliant layer. The top surface of the compliant layer is facing a semi-infinite space filled with fluid. The surface impedances Z_1, Z_{12} and Z_2 of the fluid, the compliant layer and the basic boundary, respectively, are indicated. Also indicated, are the velocities V_1 and V_2 on top and bottom surfaces of the compliant layer, respectively. An external drive, P_1, is shown applied at the interface of the compliant layer and the fluid.



b)

Fig. 2b. Equivalent circuit diagram of the model depicted in Fig. 2a showing also the possibility than an external drive, P_e, may be applied directly to the basic boundary. When $P_1 = 0$, the radiated pressure is $P_{rad} = Z_1 V_1$.

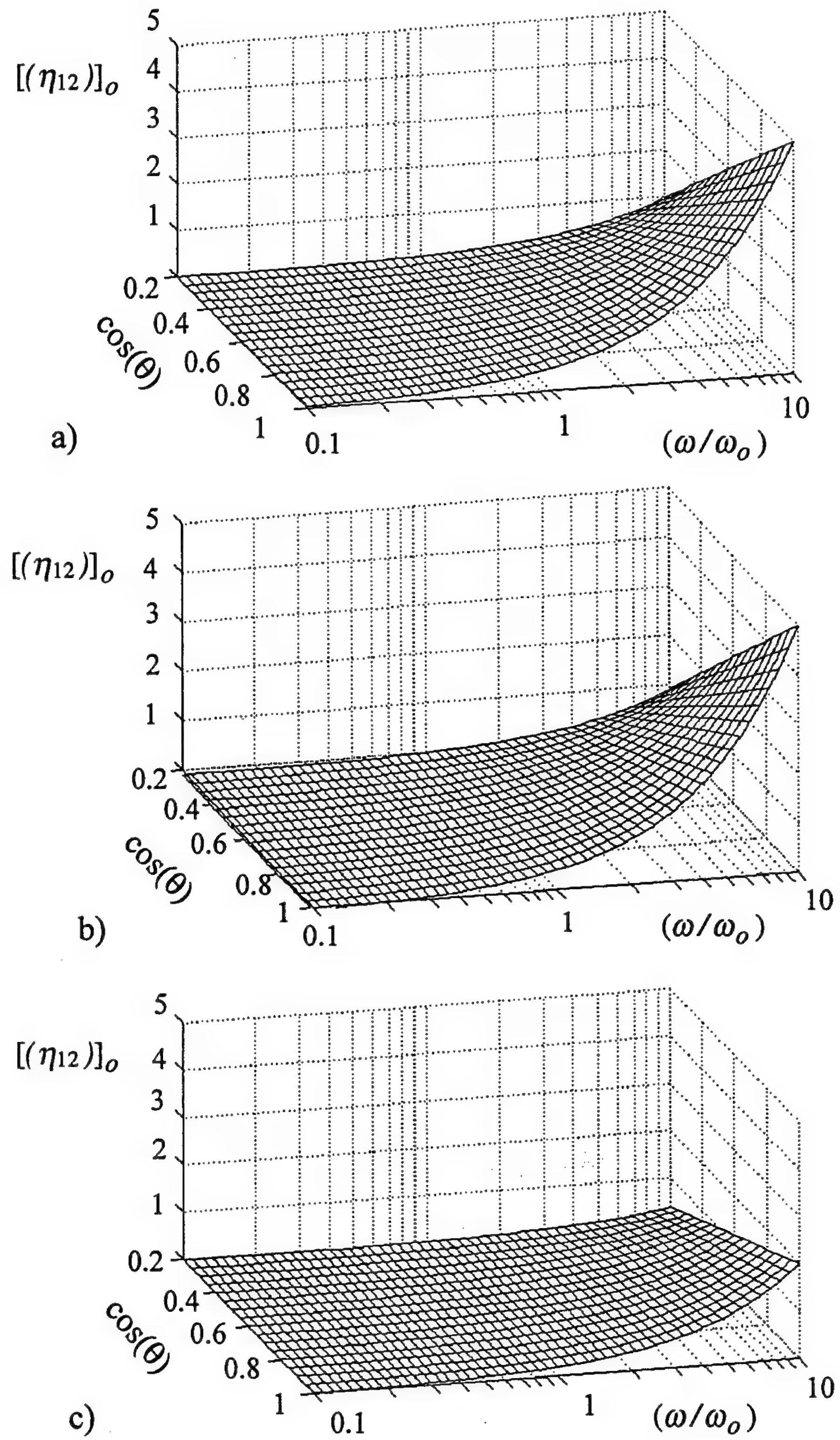


Fig. 3. The loss factor $[(\eta_{12})]_o$ as a function of (ω / ω_o) and $[\cos(\theta)]$:

a. $(\omega_o / \omega_c) = (1/20)$, $\eta_2 = 10^{-2}$

b. $(\omega_o / \omega_c) = (1/20)$, $\eta_2 = 10^{-1}$

c. $(\omega_o / \omega_c) = (1/50)$, $\eta_2 = 10^{-2}$

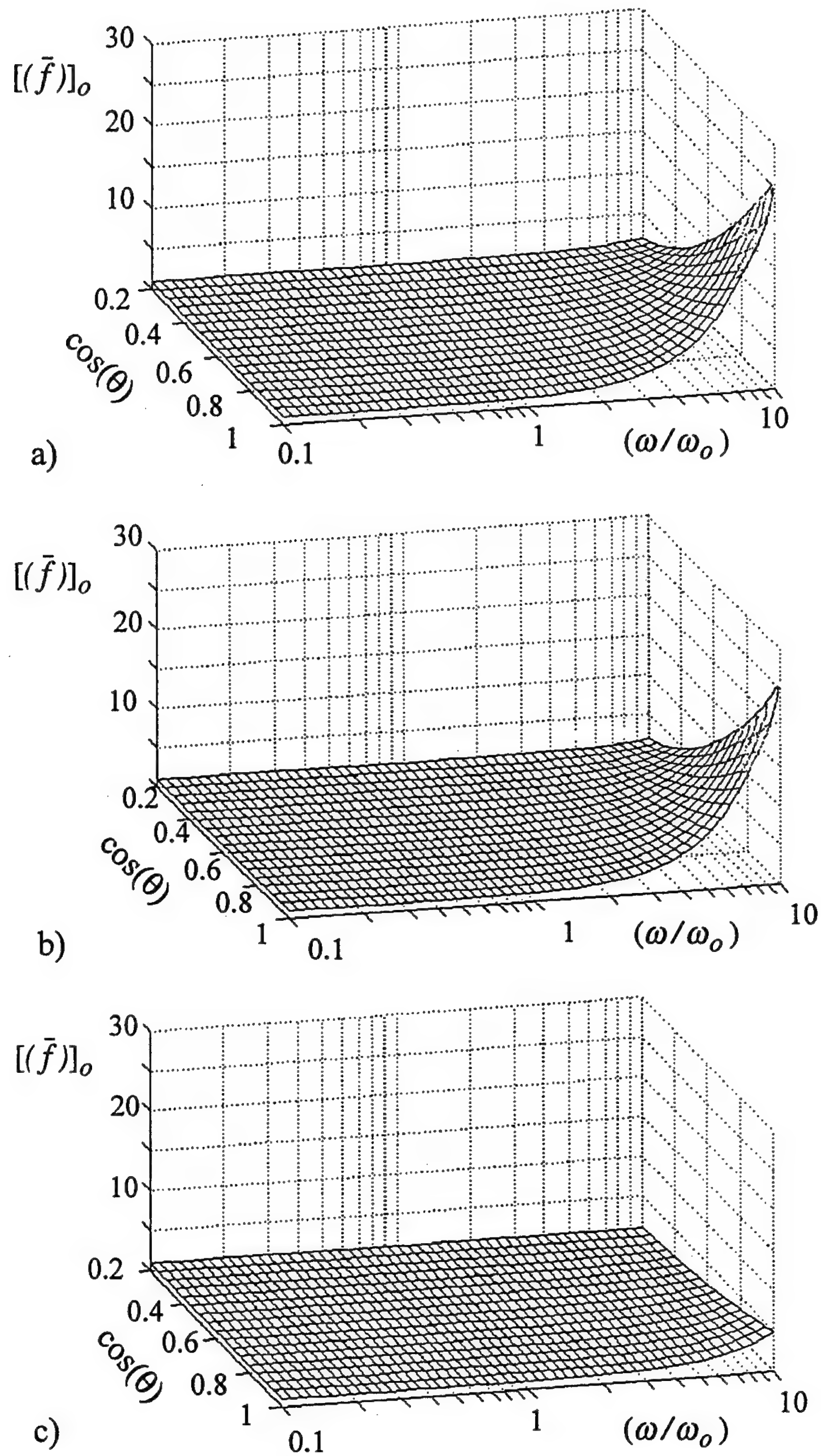


Fig. 4. The inverse modification factor $[(\bar{f})]_o$ as a function of (ω / ω_o) and $[\cos(\theta)]$:

a. $(\omega_o / \omega_c) = (1/20)$, $\eta_2 = 10^{-2}$

b. $(\omega_o / \omega_c) = (1/20)$, $\eta_2 = 10^{-1}$

c. $(\omega_o / \omega_c) = (1/50)$, $\eta_2 = 10^{-2}$

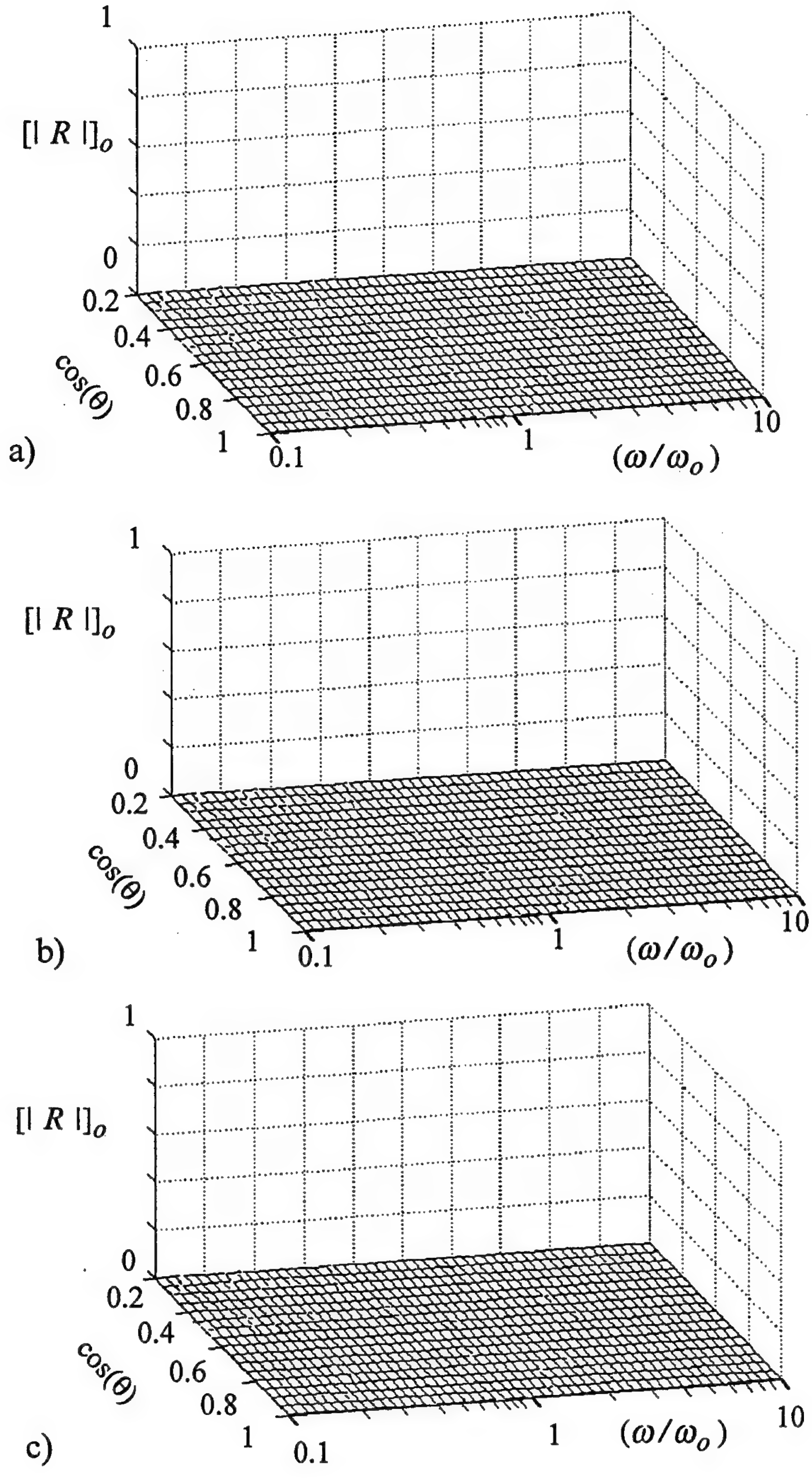


Fig. 5. The specular reflection coefficient $[|R|]_o$ as a function of (ω/ω_o) and $\cos(\theta)$:

a. $(\omega_o/\omega_c) = (1/20)$, $\eta_2 = 10^{-2}$

b. $(\omega_o/\omega_c) = (1/20)$, $\eta_2 = 10^{-1}$

c. $(\omega_o/\omega_c) = (1/50)$, $\eta_2 = 10^{-2}$

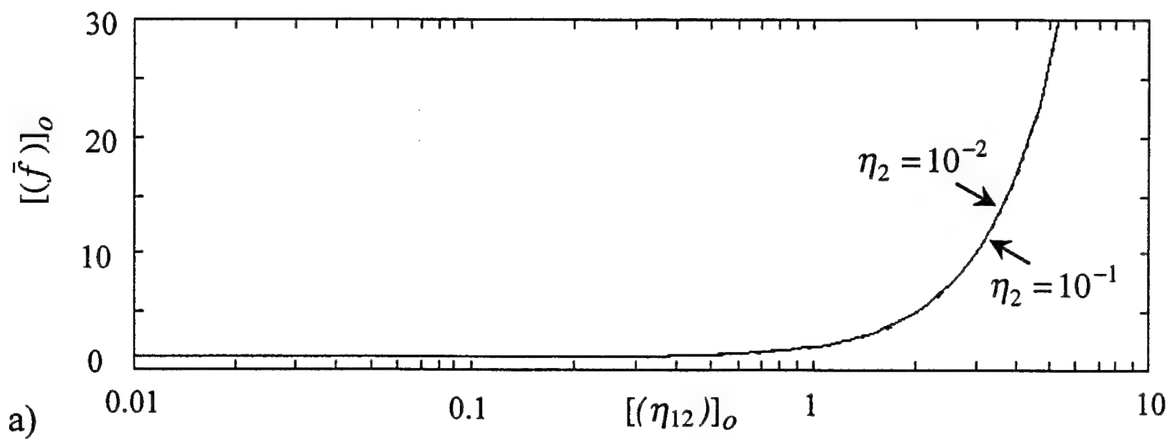


Fig. 6a. The relationship between $[(\eta_{12})]_o$ and $[(\bar{f})]_o$ curves for $\eta_2 = 10^{-2}$ and 10^{-1} are largely indistinguishable.

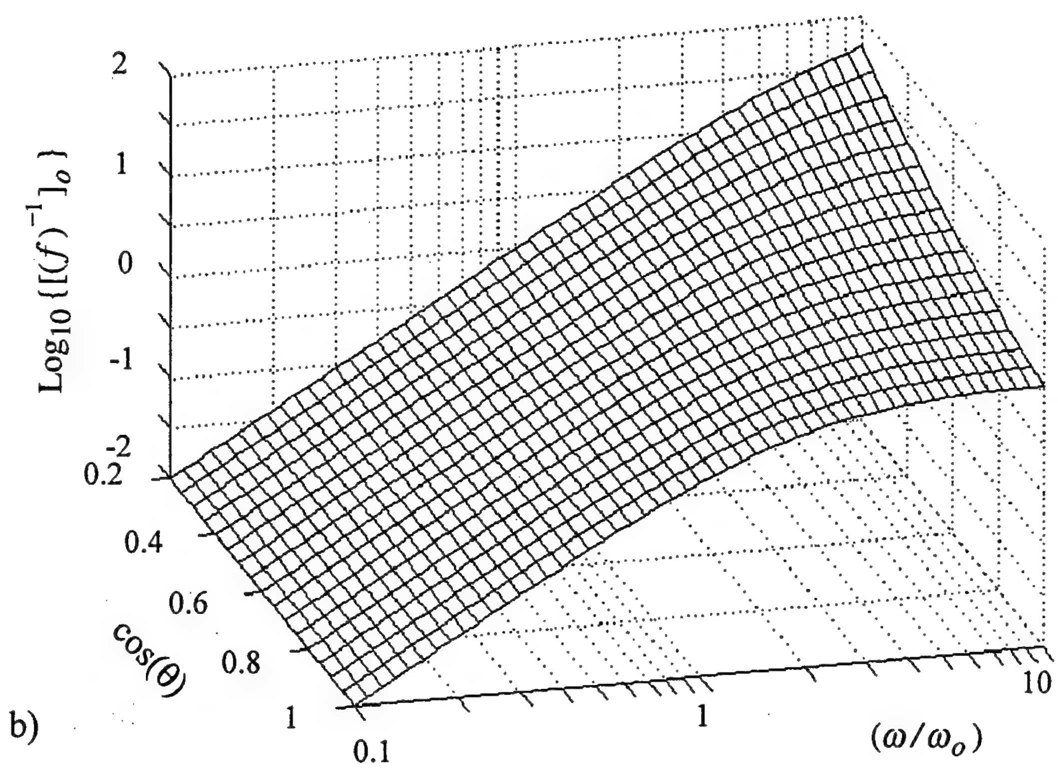


Fig. 6b. The modification parameter $[(f)^{-1}]_o$ as a function of (ω/ω_o) and cos (θ).

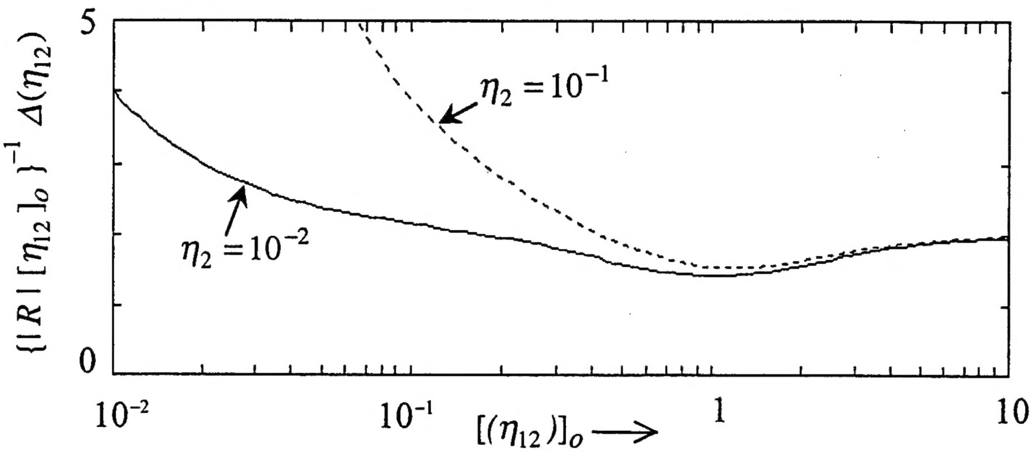


Fig. 7. The allowed normalized variations in (η_{12}) as a function of $[(\eta_{12})]_o$.

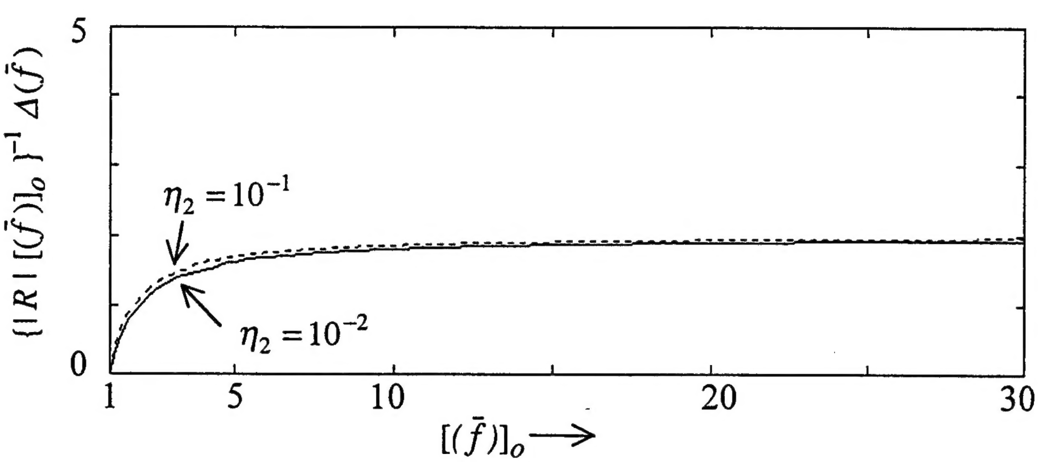


Fig. 8. The allowed normalized variations in $(\bar{f})$ as a function of $[(\bar{f})]_o$.

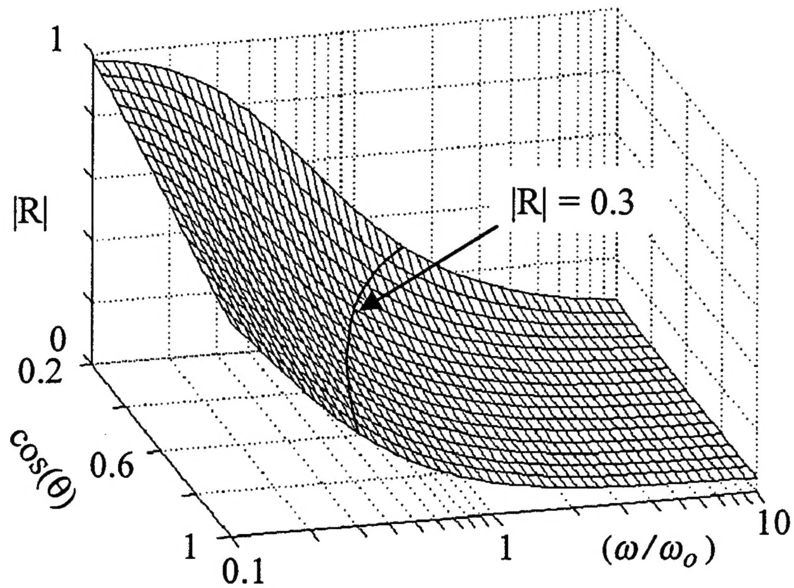


Fig. 9. The specular reflection coefficient $|R|$ as a function of (ω/ω_o) and $\cos(\theta)$ where the inverse modification factor $[(\bar{f})]_o$ is deviated by a 10%; i.e., $[(\bar{f})^{-1}]_o \ \Delta\bar{f} = \pm 0.1$

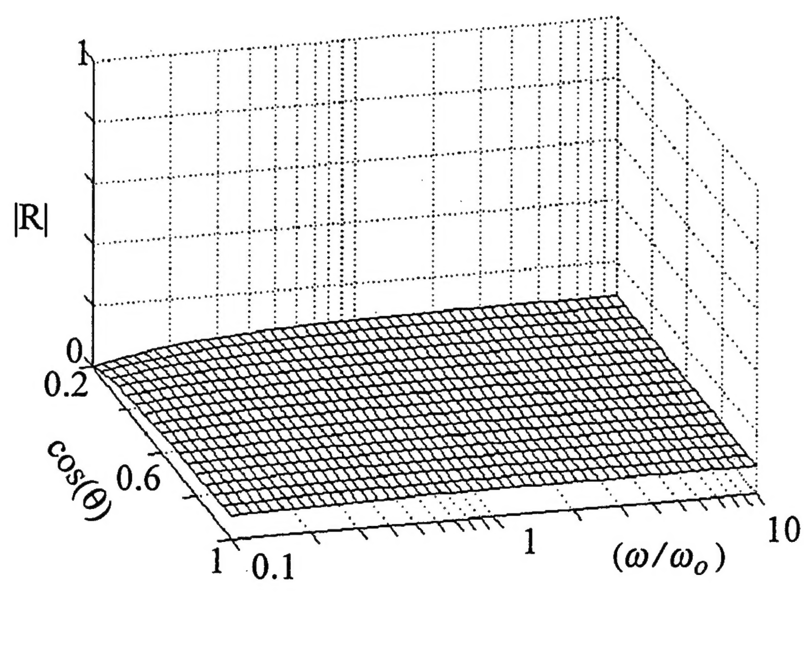


Fig. 10. The specular coefficient $|R|$ as a function of (ω/ω_o) and $\cos(\theta)$ where the loss factor $[(\eta_{12})]_o$ is deviated by 20%; i.e., $[(\eta_{12})^{-1}]_o \ \Delta\eta_{12} = \pm 0.2$

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